

AN AAD MODEL OF POINT PARTICLE AND THE PAULI EQUATION

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The classical relativistic linear AAD interaction, introduced by the author, leads in the case of weak coupling to a pointlike particle capable to be submitted to quantization via Feynman's path integrals along the line adequate to the requirements of the Pauli equation. In the discussed nonrelativistic case of the model the concept of spin is considered within early Feynman's ideas.

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1. Introduction

It is known that Feynmanian quantization is suitable for systems having a classical analogue. If such an analogue does not exist, e.g. as is the case for particles with spin, one encounters difficulties. In this context we want in the present work to illustrate the following facts: 1) the problem of spin could be attempted by renovating Feynman's spin idea [1], according to which a unit vector represents the spin germ cell in the nonrelativistic case; 2) this picture on spin could have the cause in Wigner's suggestion of the classical free point particle with internal degrees of freedom [2]; 3) such a point of view may have its deeper origin in the weak coupling model of linear interaction [3],[4],[5], in which the internal variables, products of AAD theory, adopt the status of canonically conjugate variables; 4) the model of linear relativistic interaction, even if based on the AAD theory, is able to yield its quantum versions on the levels both of Dirac [5] and Pauli equations. The model of above type, verified in [5] with the assumptions 1)-3), suggests that the relativistic variant of Feynman quantum formalism, leads to the Feynman - Gell-Mann equation [6], equivalent to the Dirac equation.

The concept of particle, used in [5] and established on the unit vector \vec{n} , which is associated with the particle internal variables, is the key tool for the Feynman non-relativistic way of quantization proposed in the present article. The model permits to evaluate the changes of this vector, involved in the given amplitude, as those potentially realizable in the classical picture. As a consequence, the standard action of the particle in the electromagnetic field can be expressed in the form of a chain of δ functions. Using such the time evolution of \vec{n} , we can determine the propagator K_S for the Pauli equation.

In this work it is also indicated that some reformulation of theory for particles with the spin 1/2 within the Feynman path integrals, combined with progressive tools of AAD theory, could shed a new light on the old question about the structure of particles. An extended formulation of old theory perhaps does not exclude yet that one can hint a new way of exploring problems as to the particle spectrum, the difference between generations of elementary particles or at least why they have no spin excited states.

In this paper we describe first briefly (Sect.2) the above weak coupling AAD model. In the context with it we utilize the postulated property of canonicity of internal variables and using their constraints we link them with the Feynman idea on the character of spin, in the strict sense of Feynman's conception of a classical spin germ cell (Sect.3) We propose then the quantization continual-integral procedure of the model, starting from the standard form of the Pauli equation, adequate to it (Sect.4).

2. Major properties of the model

The linear AAD interaction has its beginning in the papers [3],[4]. It was developed into the group form that underlies the generator procedure of constrained Hamilton dynamics [7] and [8]. It was shown [8] that the realization of the Poincaré group for the testing particle is compatible with the Lie algebra of the de Sitter group $SO(2,1)$,

proper to rotator models. The causality of this kind of interaction together with that of Wheeler-Feynman AAD electrodynamic was studied in [9] (see also [10]).

The scale of problems concerning the complete explication of linear AAD interaction in both the classical and quantum regions is clearly too wide. It can be shown, between others, e.g. that this type of interaction has necessary features, demanded by the so-called Feynman argument [11], according to which there exist theories which lead directly from their equations of motion and quantum conditions to the electrodynamic-like formula for the Lorentz force and two of Maxwell equations.

An attempt to find a picture on the nature of this interaction for weak coupling - the only region, where one traditionally can understand how quantum field theories behave - was made in [4]. It was explored that in the limit of its weak potentials, i. e. if

$$m \gg g^2 A^2; \quad m^2 \gg (\vec{\pi} - g\vec{A})^2, \quad (2.1)$$

where m is the mass of the particle, $\vec{\pi}$ its canonical momentum and A^μ the linear fourpotential defined as

$$A^\mu \equiv (\phi, \vec{A}) = g(\xi^\mu - \eta^\mu), \quad (2.2)$$

with the coupling constant g , the angular momentum $M^{\mu\nu}$ generator of the object has the form

$$M^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu + b(\xi^\mu \eta^\nu - \xi^\nu \eta^\mu). \quad (2.3)$$

In (2.3) the quantities x and p are the conventional canonical variables and ξ and $b\eta$ (b being a constant, required to hold the canonicity) are postulated to be the corresponding internal variables of the particle that is now regarded (à la Wigner; it has namely Wigner's proportion) as a free particle. Strictly speaking it should be named the canonical weak potential particle, or briefly only the WP particle. The Hamilton canonical formalism yields for both the pairs of the canonically conjugate variables the appropriate equations of motion [4], the internal variables being subjected to the constraints

$$\xi^2 = 0; \quad \eta^2 = 0. \quad (2.4)$$

Here we are obliged to respect both the validity of (2.4) and the canonicity of variables and thus to calculate with the Dirac brackets. These brackets amount

$$\{x^\mu, p^\nu\}^* = g^{\mu\nu}; \quad \{\eta'_k, \xi_i\}^* = \delta_{ik}; \quad \{\xi_k, \xi_i\}^* = \{\eta'_k, \eta'_i\}^* = 0, \quad (2.5)$$

if $\vec{\eta}' = \vec{\eta} - \frac{\eta^0}{\xi^0} \vec{\xi}$. As it was shown [4], the generators π^μ and $M^{\mu\nu}$ of the linear interaction are the products of the weak potential (WP) limit. We note that in the relation (2.3) the mechanical momentum enters in the role of the canonical one, since in the WP limit $\pi^\mu = gA^\mu$ and hence $p^\mu = \pi^\mu$. Thus the generators p^μ and $M^{\mu\nu}$ obey (2.5) or its early variant

$$\{\xi^\mu, \eta^\nu\}^* = b^{-1} \left(g^{\mu\nu} - (\xi \cdot \eta)^{-1} \xi^\nu \eta^\mu \right),$$

being next $\{\xi^\mu, \xi^\nu\}^* = \{\eta^\mu, \eta^\nu\}^* = 0$. This is a notable sign of the considered model and it can lead to serious physical implications about the character of AAD interaction at all. It is apparent at first sight that the variables ξ and η , introduced in [3], evoke to ascribe to them a statical (position) - dynamical (momentum) meaning as to the

internal variables characterizing the profile of the WP particle. Actually, they express the intrinsic fact that the dependence on the position of the particle is merely implicitly hidden in them. Next, they have (the second one multiplied by the convenient constant) the feature of canonicity a this property is linked to constraints, just those on the world cone, which become today attractive from the point of view of quantization.

3. Starting point of quantization: the classical germ cell of spin

Let us introduce into dynamics of the examined particle the unit vector, which is constructed from the one of internal components of variables ξ or η . This particle is characterized by the fixed mass m , momentum p and position x . Beside these variables we shall describe particle states by the unit vector $\vec{n} = \vec{\xi}/\xi$, where ξ is the magnitude of the vector $\vec{\xi}$. In the relativistic theory the introduction of the fourvectors ξ and η and with them also \vec{n} is demanded, if we insist on the validity of condition (2.4) and require the realization of PG of the WP particle to be transitive even for $p^2 \geq 0$. Wigner secures the transitivity of the generator (2.3) of PG only by choosing the constant b negative for the representations with $p^2 = 0$ and equal to zero for $p^2 < 0$.

In the nonrelativistic theory the physical interpretation of \vec{n} can be argued by the Feynman idea [1], expressed in connection with the classical limit of the Dirac equation. Feynman suggests the need of existence of a nonzero vector given by the ratio of spin and magnetic momentum of the particle. In the classical limit this vector is not cancelled and fulfils the equation characterizing the spin precession in an external electromagnetic field. Since in the classical limit both the quantities are equal to zero, the vector expressing the above ratio decouples from the orbital degrees of freedom and becomes an "isolated" vector without the proper physical interpretation. It represents, however, a classical "germ cell" of the spin and activates itself in the state of transition to quantum theory. ξ and η are thus in a sense the "dumb" variables, which begin to communicate only in the quantum theory. Hence there is nothing against common sense, if one admits the existence of a unit vector as an element of description of the point particle states. And so we associate its existence directly with the internal variables, to the best of our belief that this conception can have the serious physical ground, linking the spin with the weak linear interaction.

Even if one cannot yet go into whys and wherefores thorough, nevertheless we confine ourselves to the nonrelativistic Pauli equation, which will reflect the Feynman idea on spin as a unit vector and our assumption that this vector is a classical relic of the internal variables of the WP particle, discussed in Sect.2. It is obvious that the full relevance of our suggestion may be revealed in the relativistic form of this equation. Results of this kind of computations are given in [5].

As is known, quantization via Feynman's integrals is based on the definition of the action, which determines the phase of the amplitude, provided that this amplitude belongs to the appropriate path. The continual integrals for the propagator of the nonrelativistic Pauli equations were studied by Schulman [12]. In the present case, oppositely to the relativistic one [5], one can find no sufficiently simple action which would be able to generate the equation of motion for \vec{n} . Let us suppose thus that \vec{n}

obeys the equation

$$\frac{d\vec{n}}{dt} = \frac{e}{mc} \vec{n} \times \vec{H}, \quad (3.1)$$

where e is the charge of the particle and \vec{H} the strenght of external magnetic field. We suppose here that the particle owns the electric charge, of course. Eq.(3.1) together with the equation of motion for \vec{x}

$$m \frac{d^2 \vec{x}}{dt^2} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}) \quad (3.2)$$

represent the starting equations for Feynmanian quantization. The form of (3.2) indicates that in the given case we need define no action for quantization. The time evolution of \vec{n} is namely governed by the same classical law as in quantum theory. There is no external violation of evolution for \vec{n} and we are able to determine this vector precisely at any time. This is the opposite situation to the case of evolution of \vec{x} , where it holds

$$\frac{d\vec{x}}{dt} = m^{-1}(\vec{p} - \frac{e}{c} \vec{A}), \quad (3.3)$$

where \vec{A} is the vectorpotential of the external field and \vec{p} cannot be given accurately at the arbitrary time together with \vec{x} .

This consideration allows to make the conclusion that the amplitude belonging to the appropriate path is nonzero only if \vec{n} is changed along it in the classical way. It may be written down symbolically as follows

$$e^{\frac{i}{\hbar} S} \delta \left(d\vec{n} - \frac{e}{mc} \vec{n} \times \vec{H} dt \right), \quad (3.4)$$

where the Dirac delta function assures the classical equation of motion to be satisfied and next

$$S = \int_{t_1}^{t_2} \left(\frac{m}{2} \vec{v}^2 - e \tilde{A}^0 + e \frac{\vec{v}}{c} \cdot \vec{A} \right) dt \quad (3.5)$$

is the standard action of the point particle in the electromagnetic field.

Let us explain how to understand the δ function in (3.4). For this purpose let us divide the time interval $t_n - t_1$ into $N - 1$ parts of the lenght dt and denote \vec{n} at the time t_i as \vec{n}_i . The equation of motion for \vec{n} can be then written down as

$$\vec{n}_{i+1} = \vec{n}_i - \frac{e}{mc} \vec{H}_i \times \vec{n}_i dt, \quad (3.6)$$

or

$$\vec{n}'_i \cdot \vec{n}_{i+1} = 1, \quad (3.7)$$

where

$$\vec{n}'_i = \vec{n}_i - \frac{e}{mc} \vec{H}_i \times \vec{n}_i dt.$$

We see that \vec{n}'_i is the vector \vec{n}_i rotated on $\delta \vec{\varphi}$

$$\vec{n}'_i = \vec{n}_i - \delta \vec{\varphi} \times \vec{n}_i, \quad (3.8)$$

where $\delta\vec{\varphi} = (e/mc)\vec{H}dt$. Eq.(3.7) is equivalent to the original vector equation, because due to the validity $\vec{n}_i'^2 = 1$ and $\vec{n}_{i+1}^2 = 1$, one deduces $\vec{n}_i' = \vec{n}_{i+1}$. The function δ in (3.4) can be thus expressed in the chain form:

$$\delta\left(d\vec{n} - \frac{e}{mc}\vec{n} \times \vec{H}dt\right) = \delta(1 - \vec{n}_1' \cdot \vec{n}_2)\delta(1 - \vec{n}_2' \cdot \vec{n}_3)...\delta(1 - \vec{n}_{N-1}' \cdot \vec{n}_N). \quad (3.9)$$

The chain of δ functions, given in (3.9), represents the time evolution of \vec{n} as a progressive succession of infinitesimal rotations of \vec{n} .

4. Propagator

We are interested in the propagator K in the equation

$$\left(i\hbar\frac{\partial}{\partial t} - \tilde{H}\right)K(\vec{x}_2, \vec{x}_1, t_2, t_1) = i\hbar\delta(x_2 - x_1)\delta(t_2 - t_1), \quad (4.1)$$

corresponding to the Pauli nonrelativistic equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{1}{2m}(\vec{p} - ec^{-1})\vec{A}\right]^2 + e\tilde{A}_0 - \frac{e\hbar}{2mc}\vec{\sigma}\cdot\vec{H}\right]\psi. \quad (4.2)$$

In (4.1) and (4.2) \tilde{H} and $\vec{\sigma}$ represent the Hamiltonian and Pauli matrices, respectively. According to our conception of the internal vector \vec{n} the scalar propagator K_S will represent the total amplitude for the transition $\vec{x}_1, \vec{n}_1, t_1 \rightarrow \vec{x}_N, \vec{n}_N, t_N$ and may be obtained by the summation through the particular paths and time evolutions of \vec{n}

$$K_S = \int e^{\frac{i}{\hbar}S_{N1}}\delta(d\vec{n} - \frac{e}{mc}\vec{n} \times \vec{H}dt)\mathcal{D}x(t)\mathcal{D}\vec{n}(t), \quad (4.3)$$

where as is the rule

$$\mathcal{D}x(t) = d^3x_2d^3x_3....d^3x_{N-1}(2\pi i\hbar m^{-1}dt),$$

$$\mathcal{D}\vec{n}(t) = \frac{d\Omega_2}{2\pi}\frac{d\Omega_3}{3\pi}....\frac{d\Omega_{N-1}}{2\pi}$$

and $d\Omega_i$ is the solid angle appropriate to the unit vector \vec{n}_i . As emphasized above, due to the presence of δ function in the integration over $\mathcal{D}\vec{n}(t)$ only classical histories of \vec{n} give nonzero contributions. From the amplitude K_S we obtain the final spinor amplitude K , which obeys Eq.(4.1). We must, however, introduce the spinor ζ as the solution of the equation

$$\vec{n}\cdot\vec{\sigma}\zeta = \zeta \quad (4.4)$$

with

$$\zeta = \begin{pmatrix} e^{-i\varphi/2}\cos\frac{\vartheta}{2} \\ e^{i\varphi/2}\sin\frac{\vartheta}{2} \end{pmatrix},$$

where ϑ and φ are the spherical angles of \vec{n} . Then

$$K(\vec{x}_N, \vec{x}_1, t_N, t_1) = \int K_S(\vec{n}_N, \vec{x}_1, \vec{n}_N, \vec{n}_1, t_N, t_1) \zeta_N \zeta_1^+ \frac{d\Omega_1}{2\pi} \frac{d\Omega_N}{2\pi}, \quad (4.5)$$

where ζ_1 and ζ_N are the spinors appropriate to \vec{n}_1 and \vec{n}_N , respectively.

One can easily show that there is a relation between $\zeta(\vec{n}_{N-1})$ and $\zeta(\vec{n}'_{N-1})$. The spinor ζ is simply transformed as

$$\zeta' = \zeta + \frac{i}{2} \delta\vec{\varphi} \cdot \vec{\sigma} \zeta, \quad (4.6)$$

if the transformation of \vec{n} is

$$\vec{n}' = \vec{n} - \delta\vec{\varphi} \times \vec{n}, \quad (4.7)$$

being $\vec{n}' \cdot \vec{\sigma} \zeta' = \zeta'$. The integral over $d\Omega_N$ then gives

$$\int \zeta_N \delta(1 - \vec{n}'_{N-1} \cdot \vec{n}_N) \frac{d\Omega_N}{2\pi} = (1 + \frac{i}{2} \delta\vec{\varphi} \cdot \vec{\sigma}) \zeta_{N-1}, \quad (4.8)$$

and the next one

$$\begin{aligned} \int \zeta_N \delta(1 - \vec{n}'_{N-1} \cdot \vec{n}_N) \delta(1 - \vec{n}'_{N-2} \cdot \vec{n}_{N-1}) \frac{d\Omega_N}{2\pi} \frac{d\Omega_{N-1}}{2\pi} = \\ = (1 + \frac{i}{2} \delta\vec{\varphi}_{N-1} \cdot \vec{\sigma}) (1 + \frac{i}{2} \delta\vec{\varphi}_{N-2} \cdot \vec{\sigma}) \zeta_{N-2}. \end{aligned} \quad (4.9)$$

We see that in this way all of next integrals may be calculated without the only one, over $d\Omega_1$. Thus the full sequence, scanned by means of formula (4.9), will finish with the expression $(1 + \frac{i}{2} \delta\vec{\varphi}_1 \cdot \vec{\sigma}) \zeta_1$. One can easily shown that the integral over $d\Omega_1$ represents the unit matrix.

Summarizing the mentioned considerations, we finally find that

$$\begin{aligned} \int \frac{d\Omega_1}{2\pi} \int \frac{d\Omega_N}{2\pi} \int \delta\left(\frac{d\vec{n}}{dt} - \frac{e}{mc} \vec{n} \times \vec{H}\right) \zeta_N \zeta_1^+ \mathcal{D}\vec{n}(t) = \\ = (1 + \frac{i}{2} \delta\vec{\varphi}_{N-1} \cdot \vec{\sigma}) (1 + \frac{i}{2} \delta\vec{\varphi}_{N-2} \cdot \vec{\sigma}) \dots (1 + \frac{i}{2} \delta\vec{\varphi}_1 \cdot \vec{\sigma}) = T e^{\frac{i}{2mc} \int_{t_1}^{t_N} \vec{H} \cdot \vec{\sigma} dt}, \end{aligned} \quad (4.10)$$

where T is the symbol of chronological order. For the total amplitude K we obtain

$$\begin{aligned} K(\vec{x}_N, \vec{x}_1, t_N, t_1) = \int e^{\frac{i}{\hbar} S} T e^{\frac{ie}{mc} \int_{t_1}^{t_2} \vec{H} \cdot \vec{\sigma} dt} \mathcal{D}\vec{x}(t) = \\ = T \int e^{\frac{i}{\hbar} S + i \frac{e}{mc} \int_{t_1}^{t_2} \vec{H} \cdot \vec{\sigma} dt} \mathcal{D}\vec{x}(t). \end{aligned} \quad (4.11)$$

It is evident that this amplitude obeys the Pauli equation (4.2). Let us add that even the scalar amplitude K_S can be considered in the role of the propagator, if one admits

the expression $\zeta^+\psi$ to be the wave function with ψ understood as the standard spinor amplitude of initial state

$$\psi = \begin{pmatrix} \psi_i(\vec{x}, t) \\ \psi_f(\vec{x}, t) \end{pmatrix}.$$

In this case the amplitude of transition from the state ψ_i to the state ψ_f has the form

$$A_{fi} = \int K_S(\vec{x}_N, \vec{x}_1, \vec{n}_N, \vec{n}_1, t_N, t_1) (\psi_f^+ \zeta_N) (\zeta_1^+ \psi_i) \frac{d\Omega_1}{2\pi} \frac{d\Omega_N}{2\pi} d^3x_1 d^3x_N. \quad (4.12)$$

It turns out that the quantization procedure, examined in Sect.4, is possible to be extended to particles with higher spins. For instance, for complete spin s it is sufficient to replace the spinor ζ by the $2s + 1$ component quantity

$$Y_s(\vec{n}) = \begin{pmatrix} Y_{s1}(\vec{n}) \\ Y_{s1-1}(\vec{n}) \\ \vdots \\ Y_{sm}(\vec{n}) \\ \vdots \\ Y_{s-m}(\vec{n}) \end{pmatrix}, \quad (4.13)$$

where $Y_{sm}(\vec{n})$ is the spherical function. Under the infinitesimal rotations \vec{n} this function is transformed as follows

$$Y'_s = (1 + i\delta\vec{\varphi} \cdot \vec{s}), \quad (4.14)$$

where \vec{s} is the spin operator. Consequently, the integrals over $d\Omega_{\vec{n}_i}$ allow to be calculated as simply as in the case of spin 1/2. The final integral $\int Y_s Y_s^+ d\Omega$ leads also to the unit matrix.

5. Conclusion

As emphasized in Sec.2, we explore in this paper the AAD model of particle, elaborated from the linear AAD interaction [3] using its weak potential [4] (denoted workingly by the WP symbol). The WP particle exhibits features close to Wigner's model of elementary particle [2],[14]. The formal unifying both the models demanded merely to codify the WP variables ξ and η as a pair of canonical variables and give them the physical meaning of a bridge from the internal variables to spin.

The point character of a particle is more adequate to the early physical comprehension of elementariness, as accented by F. Rohrlich [13]. However, one can ask him a question, is there the possibility to construct the model of a point particle, states of which allow to ignore the Wigner condition imposed on the irreducibility of representations of the Poincaré group [2],[14] for an elementary particle? Such a possibility seems to be real in AAD theory. In the Wheeler-Feynman theory the particle pointness is regarded as the requirement for the particle to be fully described by giving its complete

worldline. Within this theory one can find cases hinting that the point particle may be correctly described without insisting at the irreducibility. Hence two above concepts - the pointness and elementariness - do not appear to be quite equivalent, the former being of the wider content.

In the AAD approach, underlain by the extended Dirac generator procedure for constrained Hamilton dynamics [7],[15] and [8] the generators of motion together with constraints, which are associated with the model, determine the equations of motion and by help of them permit to monitor the pointness. In such a way, using the AAD formalism, one obtains yet a set of other advantages for description of particles. Due to them the AAD theory of linear interaction [3] offers the model of the WP particle [4] precisely à la Wigner's pattern [2] of free particle with the internal variables ξ and $b\eta$, obeying the light cone constraints. In contradiction with Wigner's conception of elementariness, which imposes some limits on p^2 , the idea of pointness, based on AAD constraint dynamics with the same constraints, contains no such restrictions. This model allows, as was seen, to renovate Feynman's idea on spin [1], which has its nonrelativistic core in the form of a unit vector, composed of "dumb" internal dynamical variables of our model and arriving on the physics scene till in the quantum theory. Thanks to this vector, it was possible to obtain the propagator corresponding to the Pauli equation in Feynman's way of quantization.

It turns out that there exists a possibility how to associate the model of Dirac particle, discussed in [5], with the model of top as a string at low energies [16] (we are indebted to its author M. Petráš - now already died; he will be missed but not forgotten by our physics community - for useful suggestions) at a convenient mediatory role in establishing the link between strings and the spin 1/2 particles.

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